1- Write down homogeneous and inhomogeneous equations of Maxwell theory in covariant form. Relate these equations to traditional non-covariant form.

2- An extremely useful identity for the variation (in particular, the derivative) of the determinant $g := |\det g_{\mu\nu}|$ of the metric is (for more information see section 5.6 of Blau's lecture note)

$$g^{-1}\delta g = g^{\mu\nu}\delta g_{\mu\nu}\,, \qquad g^{-1}\partial_\lambda g = g^{\mu\nu}\partial_\lambda g_{\mu\nu}$$

(a) Show that this implies that covariant divergence of a vector (current) J^μ can be calculated by simple formula:

$$\nabla_\mu J^\mu = g^{-1/2}\partial_\mu(g^{1/2}J^\mu)$$

(b) Show that this implies that the covariant divergence of an anti-symmetric tensor $F^{\mu\nu} = -F^{\nu\mu}$ and the covariant Laplacian of a scalar f defined by $\Box f = g^{\alpha\beta}\nabla_\alpha\nabla_\beta f$ can be written as:

$$\nabla_\mu F^{\mu\nu} = g^{-1/2}\partial_\mu(g^{1/2}F^{\mu\nu}) \quad , \qquad \Box f = \nabla_\alpha(g^{\alpha\beta}\nabla_\beta f) = g^{-1/2}\partial_\alpha(g^{1/2}g^{\alpha\beta}\partial_\beta f)$$

3 - [Wald 4.1] Show that Maxwell's equation implies strict charge conservation.

$$\nabla_a J^a = 0$$

4- Establish

$$\Box A_b = -J_b$$

What was the gauge condition you used to reach this equation? Now try to establish

$$\Box A^\nu - R^\nu{}_{mu}A^\mu = -J^\nu \ , \qquad R^\nu{}_\mu = [\nabla_\mu, \nabla^\nu]$$

$R^\nu{}_\mu$ is representing geometry, you will get to these concepts later. Did you expect an non-minimal coupling with gravitational field?

5- A Lorentzian frame with timelike vector u is called the Comoving or Proper frame of the fluid. For n the number density in arbitrary coordinates we have $\nabla_\alpha(nu^\alpha) = 0$. Now consider a fluild that its neighbouring volume elements exert no action on each other (this fluid is called dust), the suppose flow lines are geodesics. Prove $T^{\alpha\beta} = nu^\alpha u^\beta$ satisfies conservation laws $\nabla_\alpha T^{\alpha\beta}$.