1- (a) Find the equation of motion for the following action in flat spacetime:

$$\mathcal{A}[\phi_1, \phi_2] = \int d^4x\big(- \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 - \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2 - V(\phi_1, \phi_2)\big)$$

(b) Use the minimal coupling prescription to generalize this action to curved spacetime. Then find its energy-momentum tensor.

2- Consider an Lagrangian $\mathcal{L} = \mathcal{L}(x, \dot{x}, \ddot{x})$. build the action and use variation principle to achieve equation of motion. Does it lead to the well-known Euler-Lagrange equations? this approach is referred s *Jerky Mechanics*[Goldstein]. Ponder about this and try to relate this to Lagrangian corresponding to General Relativity.

3- Consider the Einstein-Hilbert action

$$\mathcal{A} = \int_{\mathcal{M}} R\sqrt{-g}d^4x$$

Where the integral is over a $\mathcal{M}$ manifold. use the variation principle and achieve the equations of motion. You will get two parts where one part gives you the well-known Einstein tensor. Ignore the second part in calculations but write down what do you think the second part mean. You saw an equation in class corresponding to it as the *Palatini Equation*.

4 - Show that in two dimensional spacetime, the left hand side of the Einstein equation.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu}$$

vanishes identically. This means that the metric does not have any dynamics in $D = 1 + 1$, and the Einstein equations impose the constraint $T_{\mu\nu} = 0$ on the matter fields.

5- [Optional] [d'inverno 11.12] Use the variational principle approach to find the field equations of the theory (considered by A.S.Eddington)

$$\mathcal{L} = (-g)^{1/2}R^{abcd}R_{abcd}.$$