1- For perfect fluid, energy momentum tensor takes the form

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

where U^μ is the fluid 4-velocity.
(a) Show that WEC implies $\rho \geq 0$ and $\rho + p \geq 0$.
(b) Show that SEC implies $\rho + p \geq 0$ and $\rho + 3p \geq 0$. You can se that SEC does not imply the WEC.
(c) What is the physical meaning of SEC? what happens if you consider a state in which SEC is violated?

2- [Carrol 8.4] Show that the Lorenz gauge condition $\partial_\mu \bar{h}^{\mu\nu} = 0$ is equivalent to the harmonic gauge condition. This gauge is defined by

$$\Box x^\mu = 0$$

where each coordinate x^μ is thought of as a scalar function on spacetime. (Any function satisfying $\Box f = 0$ is known as an "harmonic function")

3- For a plane gravitational wave of the form $\bar{h}_{\mu\nu} = \epsilon_{\mu\nu} \exp(ik_\alpha x^\alpha)$, show that under the gauge transformation $h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu V_\nu - \partial_\nu V_\mu$, with $V_\mu = v_\mu e^{ik_\alpha x^\alpha}$ the polarization tensor transforms as

$$\epsilon_{\alpha\beta} \to \epsilon_{\alpha\beta} + i(k_\alpha v_\beta + k_\beta v_\alpha) - i\eta_{\alpha\beta} k^\gamma v_\gamma$$

4- (a) Using geodesic deviation equation achieve

$$\ddot{S}^\mu = \frac{1}{2} \ddot{h}^\mu_\sigma S^\sigma$$

where S^μ is separation (deviation) vector.
(b) "Gravitational wave is transversally polarized". Explain what does this sentence mean.